

BCS-BEC crossover in a strongly correlated Fermi gas

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We study the BCS-BEC crossover in the strongly correlated regime of an ultra-cold rotating two component Fermi gas. Strong correlations are shown to generate an additional long-range interaction which results in a modified crossover region compared to the non-rotating situation. The two-particle correlation function reveals a smooth crossover between the s -wave paired fermionic fractional quantum Hall state and the bosonic Laughlin state.

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In recent years techniques based on Feshbach scattering resonances [1] have enabled the study of pair condensation in ultra-cold Fermi gases [2, 3, 4]. For condensation to occur, one can distinguish two distinct physical mechanisms: (1) formation of bound pairs of fermionic atoms (molecules) which are composite bosons and hence undergo Bose-Einstein condensation (BEC), and (2) condensation of Bardeen-Cooper-Schrieffer (BCS) pairs in analogy with low temperature superconductivity. In separate publications [5, 6] both Eagles and Leggett argued that these scenarios were limiting cases of a more general theory, the so-called BCS-BEC crossover.

It was only recently that this crossover phenomenon was observed in rotating trap experiments. A vortex lattice generated in the molecular BEC phase was observed to persist into the BCS paired phase as the interaction is adiabatically tuned from repulsive to attractive across the Feshbach resonance [7]. Other experiments, where the rotation frequency was increased such that the degenerate gas enters the 2D regime, have led to the direct image of Tkachenko waves [8]. In this fast rotation regime the effects of strong correlations, such as those responsible for the fractional quantum Hall (FQH) effect, remain unobserved. It has been predicted that ultra-cold atomic systems can be brought into the FQH regime by rotating the trap at frequencies Ω close to the trapping frequency ω [9, 10]. In addition, non-rotating 2D gases have enabled the observation of the Berezinskii-Kosterlitz-Thouless crossover in a trap [11] and in an optical lattice [12].

In the present letter we study the BCS-BEC crossover in the strongly correlated regime. Usual mean field theory breaks down as the number of fermions becomes comparable to the number of vortices, which is essential to reach the FQH regime. We exploit the analogy with a FQH system and utilize the Chern-Simons gauge transformation [13], which enables us to study superfluidity in the strongly correlated regime by considering pairing of correlation-free composite fermions. These composite particles consist of a fermion and an even number of flux quanta. The pairing of fermions in the strongly corre-

lated regime is then studied by standard BCS mean field theory applied to these composite fermions. We show that the BCS-BEC crossover in the strongly correlated regime can be considered as a crossover between two FQH states.

We consider a two component Fermi system consisting of a balanced mixture of fermionic atoms in different hyperfine states represented by $|\uparrow\rangle$ and $|\downarrow\rangle$ confined by a 2D rotating harmonic trap. In the rotating frame, the Hamiltonian for this system in the FQH regime ($\Omega - \omega \rightarrow 0^-$) in the absence of interactions is given by

$$H = \sum_{\sigma} \int d\mathbf{r} \hat{\phi}_{\sigma}^{\dagger}(\mathbf{r}) \frac{1}{2m} [\mathbf{p} + \mathbf{A}(\mathbf{r})]^2 \hat{\phi}_{\sigma}(\mathbf{r}), \quad (1)$$

with m the mass of a fermion and $\mathbf{A} = m\omega y\hat{x} - m\omega x\hat{y}$ is analogous to the vector potential associated with the external magnetic field in the electronic FQH effect. The operator $\hat{\phi}_{\sigma}(\mathbf{r})$ annihilates fermionic atoms with spin σ at position \mathbf{r} . In order to simplify the above Hamiltonian we choose to work in a frame where the vector potential \mathbf{A} is gauged out. This is done by performing the Chern-Simons transformation by attaching gauge field $c_{\alpha}(\mathbf{r}) = -\hbar/\nu \sum_{\sigma} \int d^2\mathbf{r}' \epsilon_{\alpha\beta} \hat{\rho}_{\sigma}(\mathbf{r}') (\mathbf{r} - \mathbf{r}')_{\beta} / |\mathbf{r} - \mathbf{r}'|^2$ to each bare particle resulting in

$$H = \sum_{\sigma} \int d\mathbf{r} \hat{\phi}_{\sigma}^{\dagger}(\mathbf{r}) \frac{1}{2m} [\mathbf{p} + \mathbf{A}(\mathbf{r}) + \mathbf{c}(\mathbf{r})]^2 \hat{\phi}_{\sigma}(\mathbf{r}), \quad (2)$$

where $\hat{\phi}_{\sigma}$ is the annihilation operator and $\hat{\rho}_{\sigma}$ is the density of *composite* fermions of spin σ , and ν is the filling fraction, which is the ratio of the number of atoms to rotational flux quanta. The transformation is such that the average gauge field $\bar{\mathbf{c}}$ cancels the external field, thus $\mathbf{A}(\mathbf{r}) + \mathbf{c}(\mathbf{r}) = \mathbf{A}(\mathbf{r}) + \bar{\mathbf{c}}(\mathbf{r}) + \delta\mathbf{c}(\mathbf{r}) = \delta\mathbf{c}(\mathbf{r})$. However, we are left with gauge field fluctuations which are caused by density fluctuations. If we write the interaction part of the above Hamiltonian as $H_{\text{int}} = (1/2m) \sum_{\sigma} \int d^2\mathbf{r} \hat{\phi}_{\sigma}^{\dagger}(\mathbf{r}) [2\mathbf{p}\delta\mathbf{c} + \delta\mathbf{c}^2] \hat{\phi}_{\sigma}(\mathbf{r})$, we see that even in the absence of interactions between the bare particles, the Chern-Simons transformation gives rise to two- and three-body interactions [14].

The two-body part has been attributed to have important consequences for the formation of pairs in the electronic FQH effect [14]. We will neglect the induced three-body interaction and consider the induced two-body interaction V^{ind} , which has the form

$$V^{\text{ind}}(r) = \frac{2\hbar\omega}{\nu} \ln(r/\lambda), \quad (3)$$

for $r < \lambda$ and which we approximate by $V^{\text{ind}} = 0$ for $r > \lambda$. Here λ is the typical length scale associated with density fluctuations, and we assume that the induced interaction is washed-out beyond this range. The long-range interaction comes on top of the short-range two-body interaction, which is dominantly s -wave interaction for fermions in different spin states. We characterize the strength of the interaction via the scattering length a , that can be varied by using a Feshbach resonance. This resonant short-range interaction remains the same in the gauge transformed composite particle picture [15]. However, the additional repulsive long-range interaction can strongly modify the resonance properties, and will have the effect of lifting up the bound states in the potential and change the width of the resonance, resulting in a shift and modification of the crossover region. This can be treated systematically within the Chern-Simons composite particle picture, where we first need to understand the details of this composite particle interaction potential.

The composite fermions experience both a resonant short-range and the logarithmic long-range interaction. We solve a 2D scattering problem with long-range potential Eq. (3), where we note that even though the FQH effect exists in 2D systems, ultra-cold atomic systems under extreme rotations are in fact quasi-2D. Quasi here means that the confinement in the third dimension is strong compared to the remaining two. Hence, the interaction at short range is 3D in nature, and we use the relationship between the 2D and 3D scattering length [16] to set a boundary condition at $r = 0$. This is done by making use of a 2D contact potential [17]. We solve the 2D scattering equations as a function of relative wavenumber k and the 3D scattering length (a_{3D}). For every value of a_{3D} we can define an energy-dependent 2D scattering length $a(k)$, which is related to the scattering phase shift $\delta(k)$ via

$$\cot \delta(k) = \frac{2}{\pi} \left(\gamma + \ln \frac{ka(k)}{2} \right), \quad (4)$$

with γ the Euler constant. We now characterize the two-body interaction strength, which is a result of both interactions, via a coupling parameter related to the scattering length calculated for two particles at the Fermi energy:

$$g_{2D}(k_F) = \frac{2\pi\hbar^2}{m} \left(\ln \frac{2}{k_F a(k_F)} - \gamma \right)^{-1}. \quad (5)$$

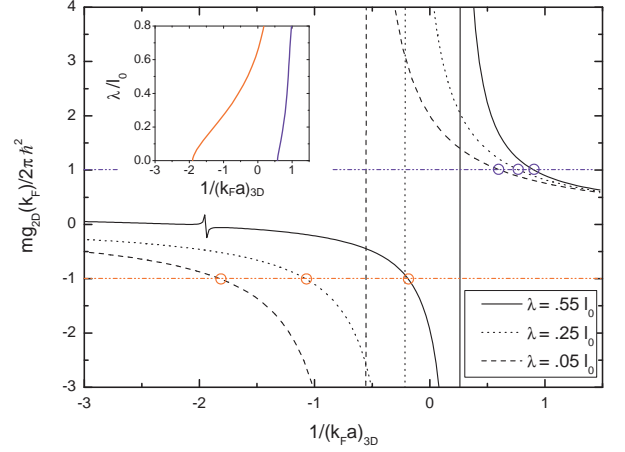


FIG. 1: (Color online) The 2D interaction strength parameter g_{2D} is plotted for different values of λ . For this (quasi-2D) system a confinement-induced resonance is observed for all values of λ . For $\lambda = .55l_0$ the atomic bound state is "pushed" out (for $1/(k_F a)_{3D} \approx -1.9$) by the rotation induced interaction, which inevitably leads to a (narrow) resonance. The inset shows the shift and narrowing of the unitarity regime for increasing λ . The ratio of axial to radial trapping frequency is here set to be 10 : 1.

In Fig.(1) this coupling parameter is given as function of the 3D scattering length, for three different values of λ . The zero-point oscillations in the third dimension can cause density fluctuations which result in gauge field fluctuations. We believe this will give a natural limit to what λ may be. On the BCS side we see that beyond $1/(k_F a)_{3D} = -1.9$ (for $\lambda = .55l_0$) no two-body bound state exists. Since this implies that no many-body pairing instability will occur [18], BCS pairs will be broken.

In the following, we use a (unitary) Chern-Simons transformation to create *dressed* composite fermions [19]. This transformation provides a direct relation between the wavefunctions of the fermions (Ψ_F) and composite fermions (Ψ_{CF}),

$$\Psi_F = \Psi_{CF} \prod_{i < j} (z_i - z_j)^{\frac{1}{\nu}} \prod_{i < j} (\xi_i - \xi_j)^{\frac{1}{\nu}} \prod_{i, j} (z_i - \xi_j)^{\frac{1}{\nu}} \times \exp \left[- \sum_k |z_k|^2 / 4 - \sum_k |\xi_k|^2 / 4 \right], \quad (6)$$

where z and ξ are scaled in units of harmonic oscillator length l_0 and represent the complex coordinate of the spin up and spin down components respectively. The wavefunction of the dressed composite fermions is not blurred by two-body correlations [20].

We will consider a system at filling fraction $\nu = 1/2$ (since this will be the experimentally most accessible regime), which means we have transformed fermionic atoms to *free* interacting composite fermions which will fill a Fermi sea. Comparison of Eq. (6) to well-known paired FQH states, e.g. Haldane-Rezayi and Moore-

Read states, shows that the pairing part of these FQH states corresponds to the wavefunction of the composite fermions. Now in the presence of some weakly attractive interaction, for instance caused by an attractive atomic interaction, these composites can form BCS-like pairs. Hence, we write the Hamiltonian for the composite fermion system in the standard BCS form and apply (standard) mean field theory. We start with the Bogoliubov Hamiltonian in diagonalized form $H_{CF} = \sum_{\mathbf{k},\sigma} E_k \gamma_{\mathbf{k}\sigma}^\dagger \gamma_{\mathbf{k}\sigma} + \text{const.}$, [21], where $\gamma_{\sigma,\mathbf{k}}^\dagger$ is the creation operator for an quasi-particle with energy $E_k = \sqrt{(\epsilon_k - \mu)^2 + \Delta_k^2}$, and ϵ_k is the single particle kinetic energy. Following Randeira *et al.* [18] we solve the number and gap equation self-consistently and find the chemical potential equals $\mu = \epsilon_F - |E_b|/2$, where ϵ_F is the Fermi energy and E_b equals the dimer binding energy in free space inferred from the energy dependent scattering length, both at zero temperature. In the low energy limit the gap function becomes a constant $\Delta = \sqrt{2\epsilon_F |E_b|}$.

Since we consider *s*-wave *spin singlet* paired fermions, the configuration space first quantized wavefunction for $2N = N_\uparrow + N_\downarrow$ composite fermions can be written as $\Psi_{CF} = \mathcal{A}(\Xi_{11'} \Xi_{22'} \dots \Xi_{NN'})$ [21], where the antisymmetrization is separately performed over up and down spins (the primed and unprimed indexes) [22], $\Xi_{jj'} = \sum_{\mathbf{k}} \frac{v_{\mathbf{k}}}{u_{\mathbf{k}}} e^{i\mathbf{k} \cdot (\mathbf{x}_j - \mathbf{x}_{j'})}$, and $\left(\frac{u_{\mathbf{k}}^2}{v_{\mathbf{k}}^2} \right) = \frac{1}{2}(1 \pm (\epsilon_k - \mu)/E_k)$. Now we are able to determine the form of the pairing wavefunctions of the (composite) fermions in the BCS and BEC regime.

To obtain a quantitative view of the crossover between the FQH states we consider a system consisting of four fermions. On the BCS side we use ‘pair’ coordinates (R_{cm}, R, r) where \mathbf{R}_{cm} denotes the center of mass position, \mathbf{R} the distance between the centers of mass of the two pairs, and $\mathbf{r}_{jj'} = \mathbf{x}_j - \mathbf{x}_{j'}$ the interparticle separation of the two fermions forming a pair. These four particles have three (equivalent) ways to form pairs which all are explicitly reproduced by Eq. (6). Since the composite fermions experience a weakly attractive interaction we know $|E_b| \ll \epsilon_F$ and the pair wavefunction of the composite fermions for $k_F r \gg 1$ is found to be $\Xi_{jj'} \propto \sin(k_F r_{jj'} - \pi/4)/\sqrt{k_F r_{jj'}}$.

Via a Feshbach resonance, we tune the *s*-wave *atomic* interaction such that the composite fermions experience a weakly repulsive interaction, i.e. $|E_b| \gg \epsilon_F$. The pair wavefunction now describes bosonic molecules since $\Xi_{jj'} = (\Delta/2\pi\epsilon_F) K_0(\kappa r_{jj'})$ exactly equals a (deeply bound) dimer state in 2D, where $\kappa = ik$. Advancing towards the BEC side means the size of the molecule (r) has become small as compared to the average inter-particle spacing ($\sim k_F^{-1}$) resulting in

$$\Psi_F = \Xi_{11'} \Xi_{22'} (r_{11'} r_{22'})^2 R^8 \exp(-R^2/4 - R_{cm}^2). \quad (7)$$

Being a degree of freedom, the coordinates $r_{11'}, r_{22'}$ can be effectively integrated out. We have obtained a state

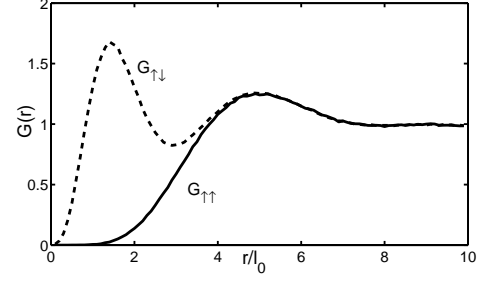


FIG. 2: The two particle correlation functions $G_{\uparrow\uparrow}(\mathbf{r})$ and $G_{\uparrow\downarrow}(\mathbf{r})$ are shown for $N_\uparrow = N_\downarrow = 100$ and $\eta = l_0$. Notice that for large r/l_0 , $G_{\uparrow\uparrow}(\mathbf{r}) - G_{\uparrow\downarrow}(\mathbf{r}) \rightarrow 0$.

consisting of composite bosons, which, form a BEC in the presence of the weak repulsive interaction. Transforming to ‘molecular’ coordinates Z , which denote the positions of the molecules in complex coordinates, results in

$$\Psi_{1/8} = \prod_{i < j} (Z_i - Z_j)^8 \exp \left[- \sum_k |Z_k|^2 / 2 \right], \quad (8)$$

which is a bosonic Laughlin $\nu = 1/8$ state [23]. The exponential contains $|Z|^2/2$ terms since the *molecular* harmonic oscillator length equals $l_0/\sqrt{2}$. Now we have seen a crossover of a paired FQH state at $\nu = 1/2$ on the BCS side to a bosonic Laughlin $\nu = 1/8$ state on the BEC side for four particles. Note that on the BEC side there are half the number of particles and the mass of each ‘elementary’ particle (molecule) has been doubled. This explains the change of the filling fraction, i.e. the ratio of number of particles to the number of flux quanta (in this case \hbar/m) from $\nu = 1/2$ to $1/8$.

This crossover between two strongly correlated states is believed to be valid for a larger number of fermions. To verify this, we have calculated the two-particle correlation function $G(\mathbf{r}_1 - \mathbf{r}_2) = \int \dots \int d^2\mathbf{r}_3 \dots d^2\mathbf{r}_N |\Psi_F|^2$ for $N_\uparrow = N_\downarrow = 100$ particles using a metropolis Monte-Carlo algorithm. Since we know the form of the paired FQH state as a function of the BCS coherence length ($\eta = \hbar^2 k_F / \pi m \Delta$) we parameterize the Monte-Carlo calculation by η . In Fig. 2, we plot both $G_{\uparrow\uparrow}(\mathbf{r})$ and $G_{\uparrow\downarrow}(\mathbf{r})$ for $\eta/l_0 = 1$. We see that $G_{\uparrow\downarrow}(\mathbf{r})$ shows a peaked behavior for small r that is absent in $G_{\uparrow\uparrow}(\mathbf{r})$. At the same time for large r , $G_{\uparrow\uparrow}(\mathbf{r}) - G_{\uparrow\downarrow}(\mathbf{r}) \rightarrow 0$ implying the existence of a sum rule special to the Haldane Rezayi like state of Eq. (6), valid throughout the region of our current interest.

Since in the $k \rightarrow 0$ limit, the *s*-wave T matrix is a smooth function of the a_{3D} *s*-wave scattering length [24], the functional form of the T matrix and hence the gap Δ near the Feshbach resonance will remain unchanged hinting a smooth crossover. The crossover behavior of the correlation function is clear from Fig. 3, which shows that as η becomes small compared to l_0 , $G_{\uparrow\uparrow}(\mathbf{r})$ gets modified continuously and tends towards a limiting form. However

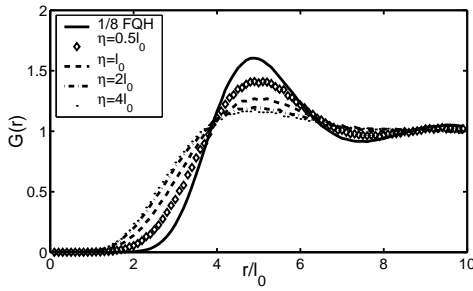


FIG. 3: In the strong pairing limit the correlation function $G_{\uparrow\uparrow}(\mathbf{r})$ has a limiting form which equals the $G(\mathbf{r})$ form of the $1/8$ FQH wavefunction Eq.(8).

the most important point to note is that the limiting form of $G_{\uparrow\uparrow}(\mathbf{r})$ is exactly that of the $G(\mathbf{r})$ for the $(1/8)$ -FQH state given by the Laughlin form in Eq. (8).

In conclusion, we have shown that the strong correlations associated with rapid rotations can cause strong modifications to the crossover, altering the width and position of the crossover. Additionally we have seen the fluctuation induced interaction can be sufficiently strong to break BCS pairs and consequently lose superfluidity. Using s -wave paired FQH wavefunctions, we have shown that the crossover is smooth and the paired FQH state of fermions smoothly goes over to $1/8$ bosonic FQH state of molecules when one goes across the Feshbach resonance such that $\eta \ll l_0$.

A detailed calculation of the crossover physics of this region will require the exact nature of the rotation-induced long-range interaction Eq. (3). Within such an treatment for instance Nozières-Schmitt-Rink calculations of the crossover region [25] can be carried out. Also these calculations can be extended to situations with p - and d - wave pairing schemes in ultra-cold Fermi gases. These scenarios, while having close resemblance with for example the $5/2$ FQH effect, will be extremely useful and will be dealt with in a future publication.

It is experimentally difficult to reach the FQH regime [26]. A promising technique is the combination of rotation with optical lattices [27], which is for instance able to reduce the number of particles per vortex.

At the same time paired FQH states such as $5/2$ are known to possess exotic non-abelian quasi-particles excitations. While existence of non-abelian statistics is the basis for topological scheme of implementing quantum logic in a quantum computer, the $5/2$ state is proved to be computationally non-universal. However, there have been proposals [28] in which this symptom can be remedied by dynamically tuning-in additional non-topological interactions. Dynamic control, while hard in the solid state configurations of the FQH effect, transitions between different FQH states like the one discussed here may be extremely useful for implementing such topolog-

ical schemes.

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